A STUDY OF THE AMOUNT OF ARITHMETIC AT THE COMMAND OF HIGH-SCHOOL GRADUATES WHO HAVE HAD NO ARITHMETIC IN THEIR HIGH-SCHOOL COURSE

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I. THE RELATIVE VALUE OF TWO METHODS USED IN DETERMINING THE POSITION OF THE DECIMAL POINT IN THE QUOTIENT IN DIVISION OF DECIMALS

The clearest evidence whether a given method in arithmetic is desirable or undesirable can be obtained by measuring the degree to which students, trained by the method in question, exhibit accuracy in their work and carry the effects of this training into their later work.

The following paper is a statement of results of a statistical study of the relative value of two methods employed to determine the position of the point in the quotient when the divisor or both dividend and divisor contain decimal places. Method A in this paper is the rule common in the older arithmetic texts and is still being taught to a considerable extent in many schools. This method may be stated as follows: "There are as many places in the quotient as those in the dividend exceed those in the divisor." Method B is the one now commonly found in the new texts and less frequently employed by teachers. It is called the Austrian method by some writers, and may be briefly stated thus: First render the divisor an integer by multiplying both dividend and divisor by 10 or some power of 10. Then proceed as with integral divisors.

This test was given as a part of a general investigation to determine how much arithmetic is actually at the command of graduates from a four-year high-school course when they enter Harris Teachers College after being away from arithmetical study for four or more years. It is given as the opening exercise in the

arithmetic course. The only directions given are: Solve the problems in the order in which they appear and place all your written work on the paper which you hand in.

The examples used in this study are:

1. $400 \div .66\frac{2}{3} + 876.16 \div .296 = ?$
2. $66.6264 \div .4 = ?$

These examples were computed by 624 different persons, 313 doing No. 1, 311 doing No. 2. Of these people, 459 had their sixth-, seventh-, and eighth-grade arithmetic in the St. Louis public schools. Of the 624, 165 had their sixth-, seventh-, and eighth-grade arithmetic, wholly or partly outside of the St. Louis public schools; 87 wholly, 78 partly. St. Louis private schools furnished 73 of the 165.

The remaining 92 are distributed among the following states and countries: Missouri, 40; Illinois, 17; Pennsylvania, 4; New York, 4; Indiana, 3; Tennessee, 3; Ohio, 2; Washington, 2; Iowa, 2; Texas, 2; Oklahoma, Kentucky, Minnesota, Alabama, Virginia, Connecticut, New Jersey, Nebraska, South Dakota, Germany, Switzerland, each 1. Of these 92 people, only 7 used the Austrian method. The first one to do so came from New York City, entering Harris Teachers College in September, 1913. None of the 73 from the St. Louis private schools used this method. This shows how generally method A was taught to within a few years ago.

The results of the tests measuring the ability to place correctly the point in the quotient are presented graphically in Figs. 1 and 2. In these figures the students tested are divided into classes and each class is recorded separately. Thus there is a class entering in January, 1909, and another entering in September, 1909, and so on to September, 1916. Each class is given its place on the horizontal.

The members of each class are now divided into two groups, those who used method A being recorded in graph A and those who used method B being recorded in graph B.

The graph is to be read as follows: In example 1, Fig. 1, there were 2 people making 2 attempts in the class of January, 1909,

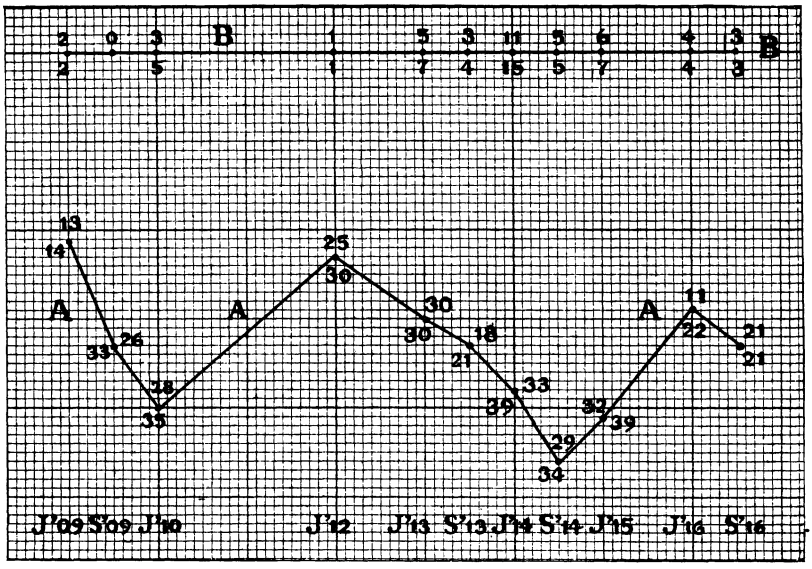
B
B
A
A
A
J'09 S'09 J'10
J'12
J'13 S'13 J'14 S'14 J'15
J'16 S'16

FIG. 1

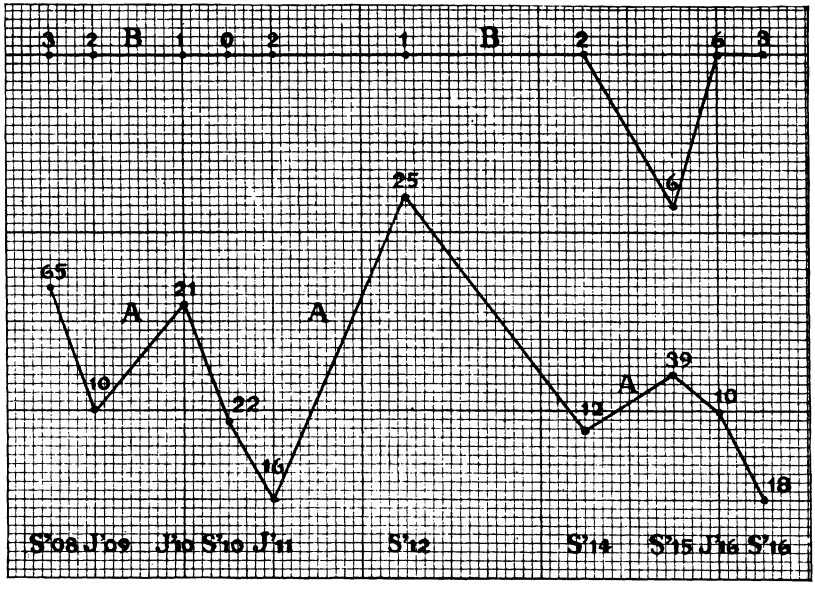
B
B
A
A
A
S'08 J'09
J'10 S'10 J'11
S'12
S'14
S'15 J'16 S'16

FIG. 2

using method B, and their score was 100. In the same class there were 13 who used method A, making 14 attempts at placing the point, and their score was 79, based on the attempts. The excess of the number of attempts over the number of individuals is explained below.

In the September, 1909, class no one used method B, and 26 used method A, making 33 attempts at placing the point, with an average score of 67, based on the attempts.

Again, in the January, 1914, class there were 11 who used method B, making 15 attempts with a score of 100. In the same class there were 33 who used method A, making 39 attempts, with an average of 62, and so on.

An inspection of example 1 shows that the student who deals with $.66\frac{2}{3}$ as a decimal must make two attempts at placing the point in the quotient, since two divisions are to be made in this example, whereas the student who converts this divisor to the common fraction $\frac{2}{3}$ will make but one attempt at placing the point. For the purposes of this paper it follows, then, that the average accuracy should be based on the attempts rather than on the number of people in the class. In Fig. 1 the numbers above the graphs represent class membership, the numbers below the graphs stand for attempts.

Likewise, an inspection of example 2, Fig. 2, shows as many attempts as there are people in the class. These graphs are to be read in the same manner as the graphs in Fig. 1. Thus, in the class of January, 1911, there were 2 people who used method B, with a score of 100, and there were 16 people who used method A, with an average score of 50.

It will be seen that the scores by method A are consistently far below those obtained by method B. The lowest score by method A was made in example 2 by the classes of January, 1911, and September, 1916, falling to an average accuracy of 50 in placing the point. The highest score by method A was also made in example 2 by the September, 1912, class, reaching an average of 84. It will be noticed that the best average class score (84, September, 1912) by method A is one point better than the poorest average class score (83, September, 1915) by method B.

The average accuracy by method A is about 66, based on 559 attempts by 507 people, and by method B about 99, based on 79 attempts by 69 people. It would appear that a method which gives on an average only 2 correct results in 3 attempts should not be permitted in the teaching of division of decimals when another method as easily taught gives practically perfect results.

It is to be noted that the graphs in the two figures represent the results of 21 classes, totaling 624 people. However, 48 of this number solved these examples by converting the decimals into common fractions, thus avoiding placing the point in the quotient. Consequently the work of these people cannot be shown in the accompanying graphs. The interesting question naturally arises, What induced these 48 people to substitute the laborious common fraction method for the easy decimal processes?

On account of the relatively small number using method B it may be thought by some readers that these people are a better selected group. As a matter of fact, in terms of their arithmetical ability determined by the complete entrance test they rank as a group about 2 per cent above the entire group of 624 people. A part of this 2 per cent apparent superiority is due to the fact that these people placed correctly the decimal point in all problems in the test involving division of decimals.

The students who employed the Austrian method in the foregoing examples also carried with them this method through their arithmetic course of one hundred hours at Harris Teachers College. An examination of all arithmetic work handed in by these people for twenty weeks showed 100 per cent accuracy in the placing of the decimal point in the quotient.

On the other hand, an examination of the work of those who continued to use method A throughout the course revealed a failure in approximately 25 per cent of the attempts.

Regardless of what the texts say, it seems in the light of the investigation described above that the Austrian method (method B) should replace the old method (method A) in all teaching of division of decimals.